Ecole Centrale de Lyon
Master in Aerospace Engineering
Fundamentals of Compressible and Viscous Flows Analysis

Final Exam - 26^{th} of January 2018

All course notes allowed. Calculator allowed.
Duration : 2 hours

Contact : *christophe.corre@ec-lyon.fr*

I - Design of a wind tunnel model generating oblique shocks (5 points)

A symmetric triangular shaped obstacle is positioned in a wind tunnel with zero incidence with respect to the incoming supersonic flow ($M_1 > 1$). The wind tunnel is assumed of height H (see Fig.1). When the oblique shock generated at the leading edge of the triangular body reaches the upper and lower walls of the wind tunnel, it is reflected so as to realign the flow in region 3 with the (horizontal) walls of the wind tunnel.
For the pressure measurements on the triangular body to be representative of flight conditions (free flow), it is crucial that the reflected shock does not interact with the body. The problem aims at computing the maximum value of the obstacle chord c (with a fixed half-angle θ characterizing the triangular body shape) such that the pressure distribution along the obstacle is not perturbed by the reflected shock. In all the problem, interaction effects between the shockwaves and the boundary layer developing in practice along the wind tunnel walls are neglected.

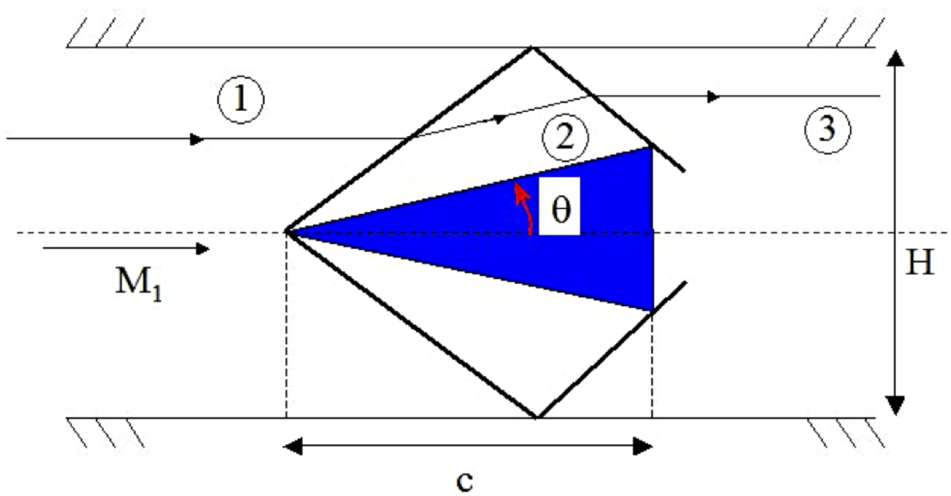

Fig. 1: Schematic view of the supersonic flow over the triangular obstacle positioned in the wind tunnel.

1) The shock angle β_1 is associated with the (incident) oblique shock separating the flow regions 1 and 2. The shock angle β_2 is associated with the reflected oblique shock separating regions 2 and 3 (see Fig. 2 corresponding to the limit case of interest where the upper reflected shock passes through the upper corner of the obstacle - only the upper part of the flow is considered for symmetry reasons).

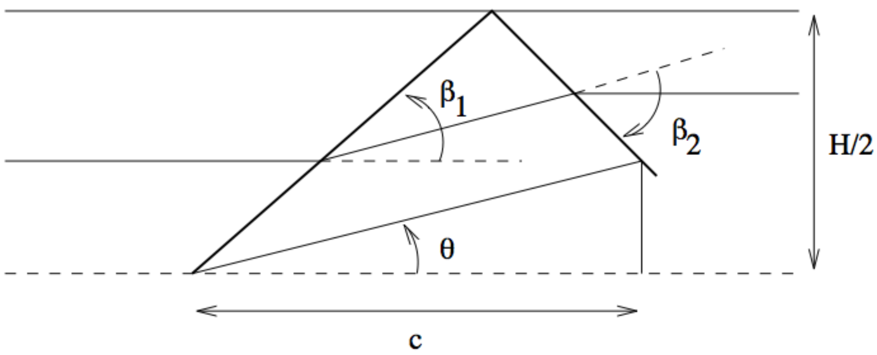

Fig. 2: Detailed view of the incident and reflected shocks.

- Compute c as a function of H, β_1, β_2 and θ. **(2 points)**

2) Assume the incoming supersonic flow is such that $M_1 = 2$ with a half-angle $\theta = 5°$ for the triangular obstacle.
- Compute c as a function of H in that case. **(3 points)**.

II - Thrust of a nozzle (10 points)

Let us consider the converging-diverging nozzle of a rocket engine. The stagnation conditions upstream of the inlet section of the nozzle are $p_0 = 20\,atm$ and $T_0 = 2000\,K$. The gas flowing through the nozzle can be considered as **a perfect gas with $\gamma = 1.2$** and $\mathbf{R = 500\,J\,kg^{-1}\,K^{-1}}$. A table giving the static to total pressure ratio p/p_0 and the local to sonic section area ratio A/A_* as a function of the local Mach number for an isentropic flow with varying section in the case of a perfect gas with $\gamma = 1.2$ is provided in Appendix 1. The throat section of the nozzle is such that $A_c = 0.1\,m^2$. The nozzle is assumed to work in a choked regime (sonic flow at the throat) in all the problem.

1) The nozzle is designed so as to be *adapted* at an altitude of $20\,km$, with temperature and pressure conditions varying with the altitude according to the standard atmosphere table provided in Appendix 2.
- Compute the area A_e of the nozzle exit section ensuring the nozzle is adapted at $z = 20\,km$ and show that $A_e = 3.24\,m^2$. **(2 points)**

2) The thrust of the rocket-engine is expressed as :

$$T = \dot{m}V_e + (p_e - p_a)A_e$$

with $\dot{m}$ the mass flowrate through the nozzle, V_e the flow velocity in the exit section, p_e the static pressure in the exit section, A_e the area of the exit section and p_a the ambient pressure outside the nozzle.

• Compute the thrust delivered at the adaptation altitude $z = 20\,km$. **(1.5 points)**
Note : to assess the likelihood of your result, consider that in similar operating conditions, the nozzles of Viking 4 or Vulcain 2 rockets develop a thrust of several hundreds of kN.

3) • Carefully explain why a straight shock appears in the nozzle, close to the exit section, when the rocket-engine is located at ground level ($z = 0$) and qualitatively describe the flow in the nozzle in that case. **(2 points)**

• Compute the thrust of the nozzle when located at ground level, assuming for the sake of simplicity the straight shock takes place exactly in the exit section. **(1.5 points)**

4) The diverging part of the nozzle is modified to ensure it is now adapted at ground level (stagnation conditions upstream of the inlet section and throat section remain unchanged).
• Compute the area of the nozzle exit section which ensures this ground level adaptation and deduce the thrust produced at ground level. **(1 point)**

• Compute the thrust of this nozzle at an altitude $z = 20\,km$ and conclude on why it is more interesting to design a nozzle adapted for ground level. **(2 points)**

III - Boundary layer development in a horn-shaped tube (5 points)

An incompressible viscous fluid with kinematic viscosity ν flows steadily in a long two dimensional horn-shaped tube (see Fig.3) with cross sectional area given by $A(x) = A_0 \exp(\beta x)$. At $x = 0$, the fluid velocity in the tube is uniform and equal to U_0. The boundary layer momentum thickness is zero at $x = 0$.

1) • Assuming no separation, determine the boundary layer momentum thickness, $\theta(x)$, on the lower boundary of the horn-shaped tube using Thwaites method. **(2 points)**

2) • Determine the condition on β that makes the no-separation assumption valid for $0 < x < L$. **(2 points)**

3) • If $\theta(x = 0)$ was nonzero and positive, would the flow in the horn be more or less likely to separate than the $\theta(x = 0) = 0$ case with the same horn-shaped tube geometry ? **(1 point)**

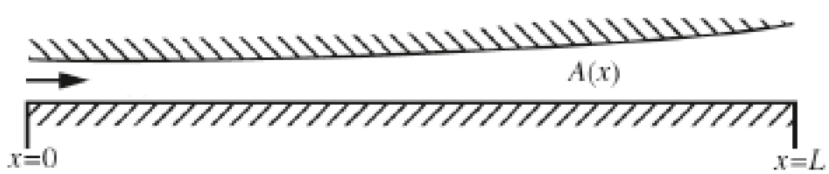

Fig. 3: Schematic view of the flow in the horn-shaped tube.

Appendix 1

Altitude z (m)	Pressure p (Pa)	Temperature T (K)
0	101325	288.16
1000	89876	281.66
2000	79501	275.16
3000	70121	268.67
4000	61660	262.18
5000	54048	255.69
6000	47217	249.20
7000	41105	242.71
8000	35651	236.23
9000	30800	229.74
10000	26500	223.26
11000	22700	216.78
12000	19399	216.66
13000	16579	216.66
14000	14170	216.66
15000	12112	216.66
16000	10353	216.66
17000	8849	216.66
18000	7565	216.66
19000	6467	216.66
20000	5529	216.66

Table 1 : Evolution of the static pressure p and the static temperature T with the altitude z for the standard atmosphere model.

Appendix 2

Mach number M	p/p_0	A/A_*
1.5	0.2959262	1.205029
1.6	0.2547195	1.296174
1.7	0.2180130	1.406962
1.8	0.1856402	1.539755
1.9	0.1573444	1.697486
2.0	0.1328103	1.883712
2.1	0.1116904	2.102679
2.2	9.3626015E-02	2.359414
2.3	7.8262754E-02	2.659828
2.4	6.5262340E-02	3.010844
2.5	5.4310001E-02	3.420533
2.6	4.5118622E-02	3.898290
2.7	3.7430879E-02	4.455013
2.8	3.1019226E-02	5.103333
2.9	2.5684860E-02	5.857848
3.0	2.1255849E-02	6.735406
3.1	1.7584676E-02	7.755420
3.2	1.4545779E-02	8.940216
3.3	1.2032876E-02	10.31544
3.4	9.9565173E-03	11.91049
3.5	8.2417605E-03	13.75901
3.6	6.8260375E-03	15.89949
3.7	5.6573064E-03	18.37580
3.8	4.6923775E-03	21.23798
3.9	3.8954976E-03	24.54293
4.0	3.2371290E-03	28.35526
4.1	2.6928943E-03	32.74830
4.2	2.2427025E-03	37.80508
4.3	1.8700159E-03	43.61945
4.4	1.5612188E-03	50.29744
4.5	1.3051123E-03	57.95852
4.6	1.0924852E-03	66.73724
4.7	9.1575994E-04	76.78490
4.8	7.6870521E-04	88.27126
4.9	6.4619014E-04	101.3866
5.0	5.4399128E-04	116.3441

Table 2 : Isentropic flow with variable section ($\gamma = \mathbf{1.2}$).