

FAMILY NAME :

First name :

The present subject copy must be returned with your exam copy. Use the provided plots to support your explanations.

1) The steepest descent (SD) algorithm is applied to the minimization of the function $f(x)$ with the graphic representation displayed in Figure 1.

• Put on Fig. 1 the points A and B corresponding to the minimum found by the SD algorithm when the initial design used by the algorithm is respectively :

i) $x^{(0)} = 1.25$, ii) $x^{(0)} = -0.25$.

Carefully justify the plotted points by explaining the working principles of the SD algorithm.

• Using the example analyzed in this question, give the key weakness of a local optimization algorithm when applied to a single-objective problem.

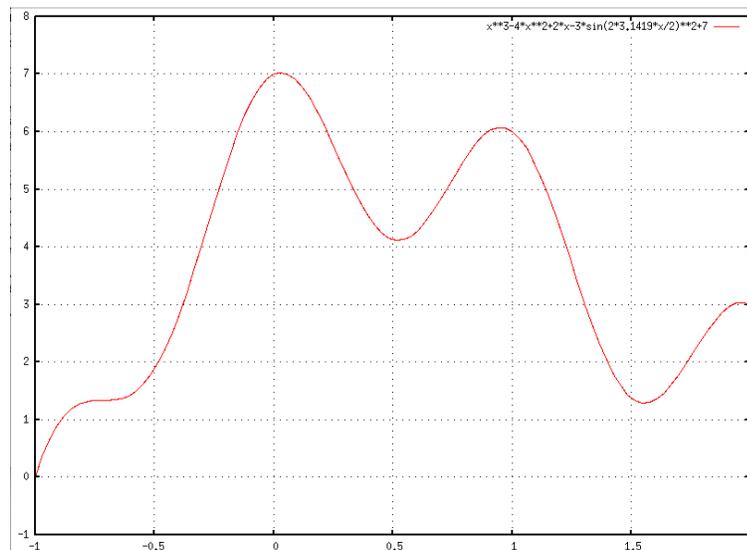


Fig. 1: Graphic display of the $f(x)$ function connected with question 1). Horizontal axis : x between -1 and $+2$ with ticks each 0.5 interval. Vertical axis : $f(x)$.

2) The BFGS algorithm is applied to the solution of problem 1) (with initial condition i) or ii)).

- Explain precisely why BFGS belongs to the family of Quasi-Newton methods.
- What makes BFGS more expensive per iteration than SD but also more efficient to reach a local or global optimum ?

3) Let us consider the SIMPLEX method in its basic 1962 formulation, that is making use of the reflection operator only. It is applied to the maximisation of the function $f(x, y) = x^2 + 3xy + 2y^2 + 1$. Assume the initial simplex corresponds to the three designs plotted as red circles in Fig. 2.

- Draw in Fig. 2, explaining carefully how you proceed, the simplex associated with 3 successive iterations of the algorithm run from this initial simplex.

4) You are looking for the shape of an airfoil profile maximizing the lift-to-drag ratio f of this

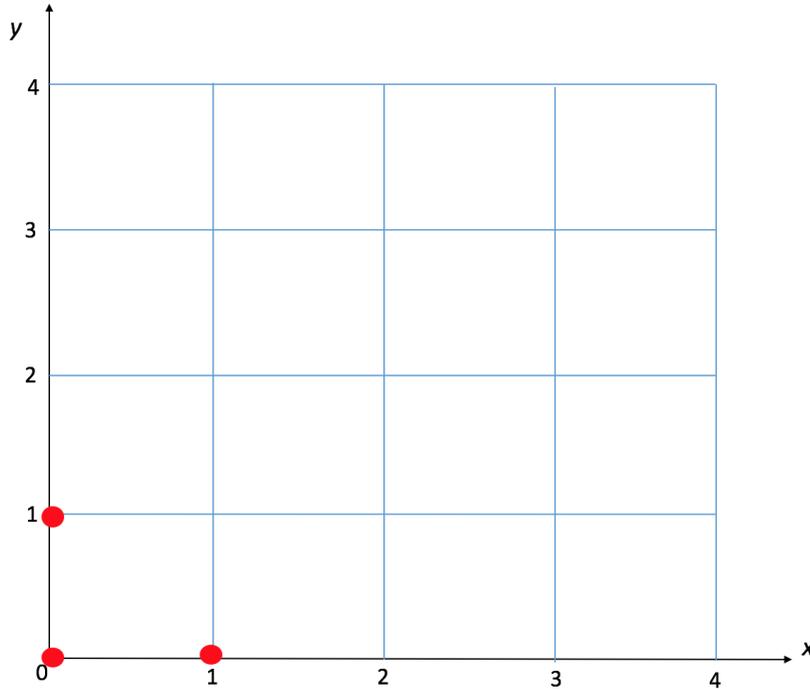


Fig. 2: Simplex evolution of the Spendley *et al.* algorithm applied to the maximization of the f function described in question 3).

airfoil in cruise conditions. The airfoil geometry is described by 10 shape parameters which define the design parameters vector \underline{x} . A CFD loop is at your disposal with the values of the 10 shape parameters as input parameters and the lift-to-drag ratio as output quantity (objective), computed for the considered operating point that is the cruise regime. Each calculation of the objective function for a given set of parameters costs 10 minutes of computations. Besides, an optimizer relying on the BFGS algorithm is also at your disposal.

- Describe in detail the strategy you plan to follow in order to apply BFGS to the problem under study, when you wish to minimize the cost of the optimization. Give (and justify) an estimate of the cost for each iteration of the optimization algorithm.

5) Let us go back to the problem described in 4) but considering now two distinct operating points (flow conditions) for which we wish to maximize the lift-to-drag ratio. Let us denote f_A and f_B the lift-to-drag ratio computed respectively for the operating point A and the operating point B . The CFD loop is adapted so as to provide these 2 values of the lift-to-drag ratio for a given set of design parameters \underline{x} . The only optimizer at your disposal remains the one used in 4) which relies on the BFGS approach.

- Explain how the available tools can be adapted to the simultaneous maximization of f_A and f_B .
- Give the limitations of the proposed approach.

6) You have now at your disposal an optimizer which relies on the NSGA optimization method (Non-dominated Sorting Genetic Algorithm) in order to solve the problem described in 5). Figure 3 displays the population of 6 individuals computed at a given generation of the algorithm. It is reminded NSGA gives the value "population size" to the fitness function for designs with a dominance rank equal to 1 then decreases this value with a quantity ϵ for the designs belonging to every new dominance rank. Assume here $\epsilon = 1/2$. The NSGA algorithm also uses a sharing function to

compute the fitness function for individuals belonging to the same local Pareto front. In order to simplify the application of the method, the sharing strategy will not be taken into account in the present calculation of the fitness function.

- Give the fitness value for each design of the population plotted in Fig.3, carefully explaining how you compute these values - you can use Fig. 3 to provide useful information.

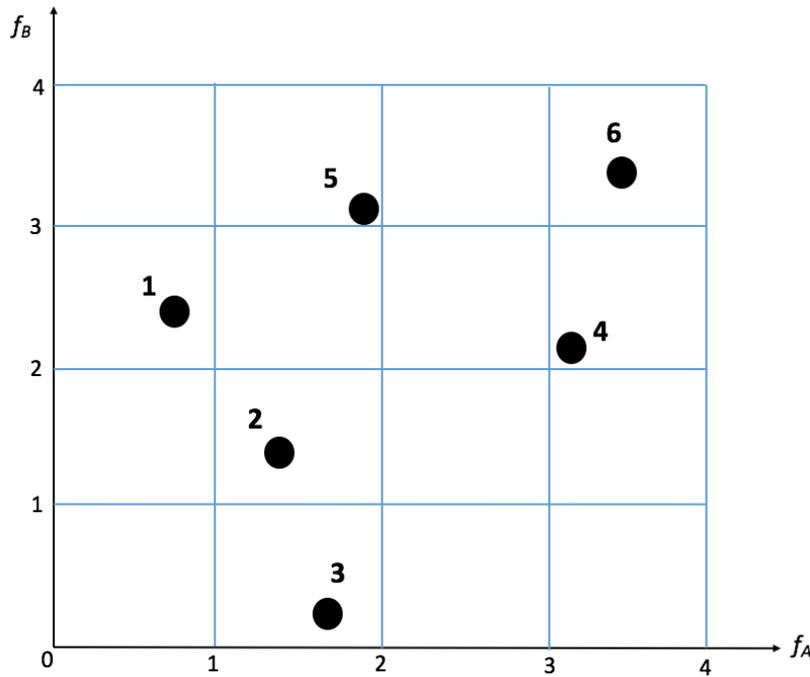


Fig. 3: Population of the 6 individuals considered in question 6).

7) A genetic algorithm is said to be a semi-stochastic optimization algorithm because it depends on processes or operators which are only partially random.

- Describe the selection process for the genetic algorithm using a proportional selection operator studied during the lectures and explain why this selection process is only partly stochastic.
- Detail the stochastic factors in the crossover and mutation operators of the genetic algorithm studied during the lectures.

8) One of your co-workers is interested in the simultaneous maximization of two objective functions f_1 and f_2 . The results he / she obtained using the NSGA genetic algorithm are displayed in Figure 4. He / she also reported on this picture the solutions provided by the SIMPLEX algorithm applied to the maximization of $\omega_1 f_1 + \omega_2 f_2$ for different choices of (ω_1, ω_2) .

- Which recommendation(s) would you make to your co-worker for the next steps to take in his/her analysis of this optimization problem ?

9) You wish to identify the various local maxima of the $f(x)$ function plotted in Fig. 5.

- Explain how the selection operator of the simple genetic algorithm must be modified to allow this detection of multiple optima. Use Fig. 5 to support your answers.

10) You wish to maximize an expensive objective function f , depending on 2 design parameters x and y . Using most of your available computational budget, you have produced a set of 10 observa-

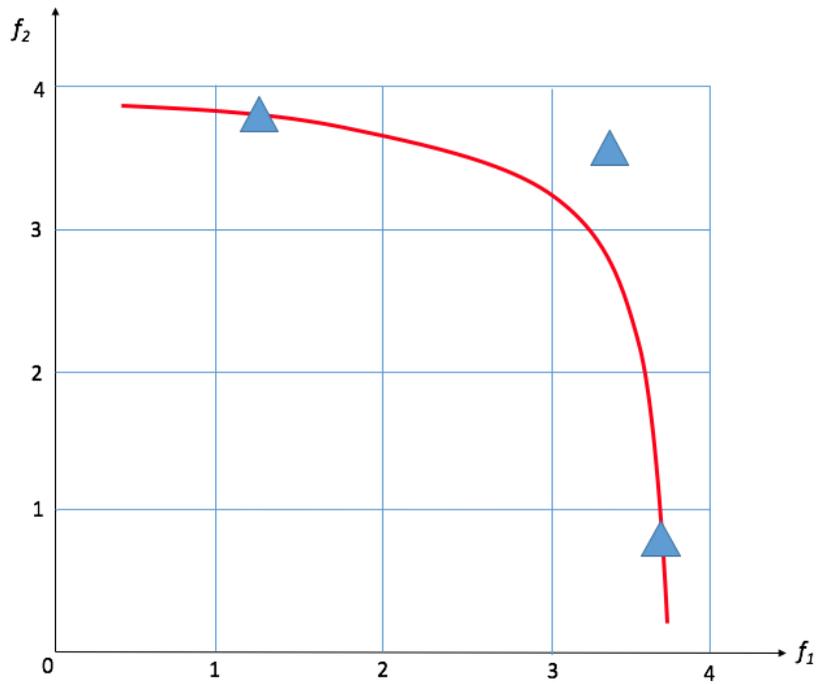


Fig. 4: Computed solutions for the simultaneous maximization of the f_1 and f_2 functions (see question 8)). Red line : Pareto front identified by the NSGA algorithm such as used by your co-worker. Blue triangles : solutions of the preference-based optimization using 3 different combinations of weights.

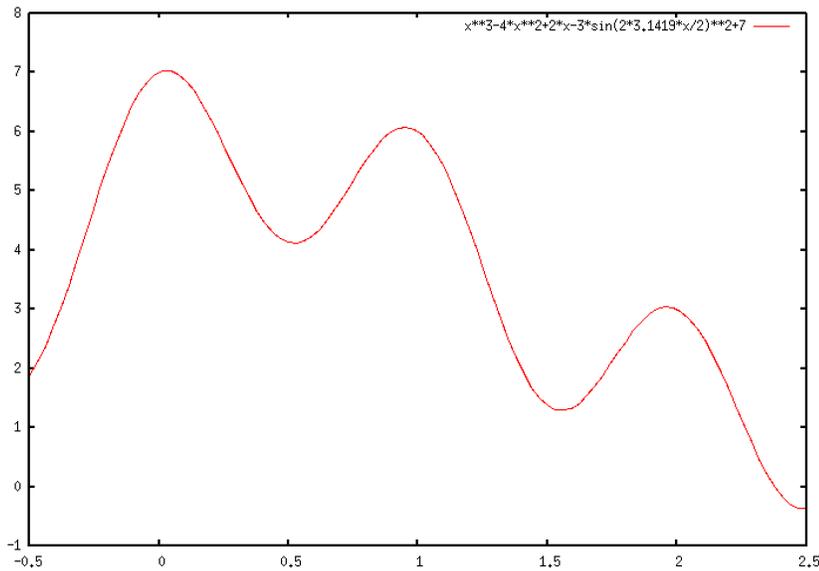


Fig. 5: Graphic display of the $f(x)$ function associated with question 9). Horizontal axis : x . Vertical axis : $f(x)$.

tions for f and decide to use it to build a polynomial surrogate function \hat{f} , assumed of order 2.

- Explain precisely how the surrogate function \hat{f} is computed.
- Describe the steps of the surrogate-based optimization you intend to perform, knowing you cannot afford more than 3 more observations for f .