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Effect of a weak polar misalignment of the magnetic field on the stabilization of the Hadley flow

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Abstract

We study the stability of an infinite differentially heated liquid metal layer bounded by two horizontal walls and submitted to an external magnetic field subjected to a slight polar deviation with respect to the initially selected direction. The effects of the weak deviation on the stability characteristics of the two-dimensional stationary and three-dimensional oscillatory instabilities which can develop in such a liquid layer are investigated. The flow exhibits some interesting and unexpected stability characteristics. The effect of the deviation is either stabilizing or destabilizing depending essentially on the initially selected magnetic field orientation: longitudinal, transverse or vertical. In general, when the deviation angle increases the critical values of the Grashof number, the wave numbers and the frequency deviate from the asymptotic behaviours observed for large values of the Hartmann number, $Ha > 20$, in the case without deviation. New features and behaviours are found for some orientations of the magnetic field.

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1. Introduction

The study of convection in a horizontal metallic liquid layer heated from the side is of great interest in many research fields and engineering applications. Among them we can mention material processing technologies and especially the Bridgman crystal growth technique. In this system, the temperature gradient is orthogonal to the gravity, and then convection (Hadley circulation) arises for any value of the temperature difference. Previous studies (see e.g. Refs. [1–3]) have shown that when the temperature gradient exceeds some critical value, spontaneous oscillations of velocity and temperature appear in the melt. The impact of these oscillations on the crystal quality was pointed out by means of experimental, theoretical and numerical investigations [4–7]. The pioneering experimental work achieved in this field is due to Hurle [8], who showed that temperature oscillations in molten metals are responsible for the appearance of striations in melt-grown crystals.

Among the techniques proposed for the improvement of the crystal quality, the use of a constant magnetic field appears as an interesting way to avoid the appearance of oscillations in the melt and consequently to ensure the achievement of striation-free crystals. Hurle et al. [1] have shown experimentally that a horizontal transverse constant

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magnetic field delays the onset of temperature oscillations in molten gallium. The approximate solutions of the linear stability analysis equations given by Gill [9] confirm the results of Hurle et al. [1] for the case without magnetic field. More recently, the work of Gill [9] was extended by Kaddeche et al. [10–12] to the case with a magnetic field by means of theoretical [10,11] and numerical calculations [12]. The results obtained by these authors show that the vertical magnetic field is the most efficient to stabilize the Hadley flow whereas the horizontal directions of the field are significantly less effective at damping instabilities, in agreement with the experimental results of Hof et al. [13]. Furthermore, for these directions of the magnetic field, the theoretical and numerical thresholds compare within a factor two to three with those obtained experimentally in Ref. [13]. Priede and Gerbeth [14,15] carried out numerical computations where they consider the effect of both vertical and coplanar magnetic fields on the stability of a fluid layer subject to thermocapillary forces. All these results [10–15] show the importance of both direction and strength of the magnetic field on the stabilization process.

This work is motivated by the interest to estimate the effect of the magnetic field in a practical situation. Since the experimental adjustment of the magnetic field direction is difficult to achieve perfectly, the effects of a weak deviation of the magnetic field with respect to its initially selected direction on the stability characteristics of the two-dimensional stationary and three-dimensional oscillatory instabilities which can develop in such a system, are investigated. A particular practical aspect among others, motivating our interest in the impact of a slight deviation affecting the polar orientation β of the magnetic field compared to the perfectly vertical direction ($\beta = 0^\circ$) and the two main horizontal directions ($\beta = 90^\circ$ with $\alpha = 0^\circ$ or $\alpha = 90^\circ$), is the fact that these directions are the most used directions for damping both convection and instabilities in the Bridgman configuration [13,16]. One should expect that the unavoidable deviation can have some consequences on the stabilization efficiency of the applied magnetic field.

2. Mathematical model

We consider an infinite horizontal metallic liquid layer of thickness H bounded by two horizontal rigid plates and subjected to a uniform and constant magnetic field $\overrightarrow{B_0}$. The undisturbed fluid motion $(\overrightarrow{V_0}, P_0, T_0, \Phi_0)$ is entirely driven by the horizontal temperature gradient, $\nabla\widetilde{T} = \Delta T/L$, imposed by the heating facility ($\Delta T = T_1 - T_0$ where T_1 and T_0 are temperatures of the hot and cold lateral vertical interfaces separated by a distance $L \gg H$). The fluid is considered as Newtonian, electrically conducting and obeying the Boussinesq law: $\rho = \rho_0(1-\beta(T - T_0))$. The horizontal boundary walls are considered as electrically insulated. Referring to Moreau [17], in most magnetohydrodynamics laboratory experiments using liquid metals, the magnetic Reynolds number is very small and one can neglect the induced magnetic field $\overrightarrow{b}$ compared to the applied external magnetic field $\overrightarrow{B_0}$.

To investigate the linear stability of the basic flow, solution of the stationary problem, we consider the evolution of an infinitesimal perturbation $(\overrightarrow{v}, p, \theta, \phi)$ of the velocity, pressure, temperature and electric potential, respectively. Such a perturbation is superimposed to the basic flow and its evolution is achieved through the linearized system of equations based on the Navier–Stokes equations, coupled to mass, energy and electric charge conservation equations. If we consider H, H^2/ν, ν/H, $\varrho_0\nu^2/H^2$, $\nabla\widetilde{T}H$ and νB_0, as reference quantities for length, time, velocity, pressure, temperature and electric potential, respectively, the linearized equations could be written as

$$\nabla.\overrightarrow{v} = 0, \tag{1}$$

$$\frac{\partial\overrightarrow{v}}{\partial t} + (\overrightarrow{V_0}.\overrightarrow{\nabla})\overrightarrow{v} + (\overrightarrow{v}.\overrightarrow{\nabla})\overrightarrow{V_0} = -\overrightarrow{\nabla}p + \nabla^2\overrightarrow{v} + Gr\theta\overrightarrow{e_z} + Ha^2\overrightarrow{j} \times \overrightarrow{e_{B_0}}, \tag{2}$$

$$\frac{\partial\theta}{\partial t} + \overrightarrow{V_0}.\overrightarrow{\nabla}\theta + \overrightarrow{v}.\overrightarrow{\nabla}T_0 = \frac{1}{Pr}\nabla^2\theta, \tag{3}$$

$$\nabla^2\phi = \overrightarrow{e_{B_0}}.(\overrightarrow{\nabla} \times \overrightarrow{v}), \tag{4}$$

$$\overrightarrow{j} = -\overrightarrow{\nabla}\phi + \overrightarrow{v} \times \overrightarrow{e_{B_0}}, \tag{5}$$

with

$$\overrightarrow{e_{B_0}} = \overrightarrow{B_0}/B_0 = \cos\alpha\sin\beta\,\overrightarrow{e}_x + \sin\alpha\sin\beta\,\overrightarrow{e}_y + \cos\beta\,\overrightarrow{e}_z. \tag{6}$$

The dimensionless numbers appearing in Eqs. (2)–(3) are the Grashof number $Gr = g\beta\nabla\widetilde{T}H^4/\nu^2$, the Prandtl number $Pr = \nu/\kappa$ and the Hartmann number $Ha = B_0H\sqrt{\sigma_e/\varrho_0\nu}$, respectively. In the expression of the external magnetic field (6), α and β are the azimuthal and the polar angles, respectively, as depicted in Fig. 1. The basic flow $(\overrightarrow{V_0}, P_0, T_0, \Phi_0)$ is a stationary parallel flow $(\overrightarrow{V_0} = (U_0(z), 0, 0))$ solution of the Navier–Stokes equations

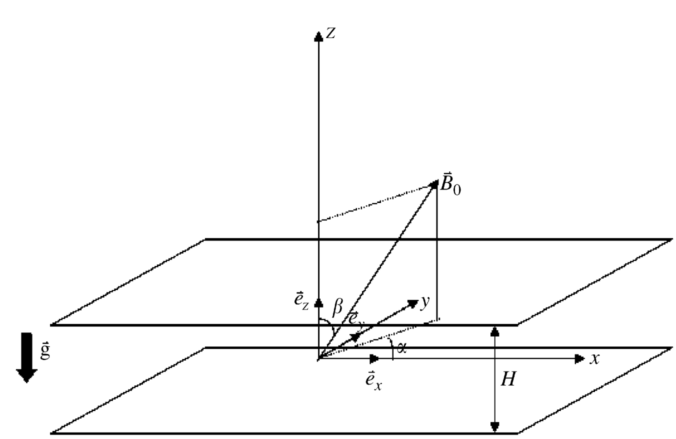

Fig. 1. Studied configuration.

coupled to both energy conservation equation and continuity equation for the electric current density vector. These equations can be reduced to the following system:

$$\frac{d^3U_0}{dz^3} - (Ha\cos\beta)^2\frac{dU_0}{dz} = Gr, \tag{7}$$

$$\frac{d^2\Phi_0}{dz^2} = \sin\beta\sin\alpha\frac{dU_0}{dz}, \tag{8}$$

$$\frac{d^2T_0}{dz^2} = Pr\ U_0. \tag{9}$$

An analytical solution of the system (7)–(9) can be easily derived for thermally conducting conditions at the rigid horizontal boundaries. The following expressions for the velocity and temperature profiles are thus obtained:

$$U_0(z) = \frac{Gr}{(Ha\cos\beta)^2}\left(\frac{\sinh(Ha\cos\beta z)}{2\sinh(Ha\cos\beta/2)} - z\right), \tag{10}$$

$$T_0(x,z) = x + \frac{Pr\ Gr}{(Ha\cos\beta)^2}\left[\frac{\sinh(Ha\cos\beta z)}{2(Ha\cos\beta)^2\sinh(Ha\cos\beta/2)} - \frac{z^3}{6} + \left(\frac{1}{24} - \frac{1}{(Ha\cos\beta)^2}\right)z\right]. \tag{11}$$

The perturbation is considered as a normal mode. It can then be written as $(\vec{v}, p, \theta, \phi) = (\vec{v}(z), p(z), \theta(z), \phi(z))\, e^{i(hx+ky)+\omega t}$ where h and k are the wave numbers in the x and y directions, respectively, and ω is a complex pulsation. The linearized equations (1)–(5) mentioned above can be transformed into an eigenvalue problem, namely $LX = \omega MX$, where $X = (\vec{v}(z), p(z), \theta(z), \phi(z))$, L is a linear operator depending on h, k, Gr, Pr, Ha, α and β, and M is a constant linear operator. Such an eigenvalue problem is solved using the spectral Tau–Chebyshev method [12]. From the thresholds $Gr_0(h, k, Pr, Ha, \alpha, \beta)$ for which an eigenvalue has a real part equal to zero whereas all the other eigenvalues have negative real parts, the critical Grashof number Gr_c is obtained after a minimization procedure with respect to h and k:

$$Gr_c = \operatorname*{Inf}_{(h,k)\in\mathbb{R}^2} Gr_0(h, k, Pr, Ha, \alpha, \beta). \tag{12}$$

3. Results

In the absence of magnetic field, it is well known that the Hadley flow becomes unstable when the horizontal temperature gradient exceeds some critical value [12,18]. For liquid metals, two types of instability can occur depending on the value of the Prandtl number. For liquid layer with conducting horizontal rigid boundaries, two-dimensional transverse stationary instabilities are the dominant modes for weak Pr, whereas the longitudinal oscillatory instabilities prevail for higher values of the Prandtl number. In previous works, Kaddeche et al. [10–12] have shown that the vertical magnetic field is more efficient in stabilizing the two-dimensional instabilities ($Gr_c \sim \exp(Ha^2)$) than the three-dimensional instabilities ($Gr_c \sim Ha^2$). Among all the horizontal magnetic field orientations α, the longitudinal one ($\alpha = 0°$) is the most efficient to delay the appearance of the two-dimensional instabilities and the transverse one ($\alpha = 90°$) is the most efficient to delay the appearance of the three-dimensional instabilities as discussed in Kaddeche et al. [19]. For such directions of the magnetic field, these authors have derived asymptotic scaling laws governing the variation of the critical Grashof number as a function of the Hartmann number, namely $Gr_c \sim Ha$ for $Ha > 20$.

However, during a laboratory experiment, it is difficult to achieve an accurate orientation of the magnetic field and a slight deviation is often unavoidable. In this study, we consider the problem of stabilizing the two-dimensional transverse modes and the three-dimensional longitudinal modes by an external magnetic field slightly deviated with respect to its initially selected direction. The selected directions are the longitudinal direction ($\alpha = 0°, \beta = 90°$), the transverse direction ($\alpha = 90°, \beta = 90°$) and the vertical direction ($\beta = 0°$). For these three directions, the considered values of the deviation $\lambda = 90° - \beta$ for the case of a longitudinal or a transverse magnetic field and β for the case of a vertical magnetic field, do not exceed 5°.

3.1. *Effect of a weak polar deviation on the transverse instabilities*

3.1.1. *Weak deviation with respect to a longitudinal magnetic field*

In order to estimate the effects of a weak polar deviation from the longitudinal direction ($\alpha = 0°, \beta = 90°$) on the threshold characteristics, we carried out a series of numerical calculations for $1° \leqslant \lambda \leqslant 5°$. In Fig. 2 are plotted the neutral stability curves as a function of Ha for various values of λ. From the figure, we can clearly notice that the

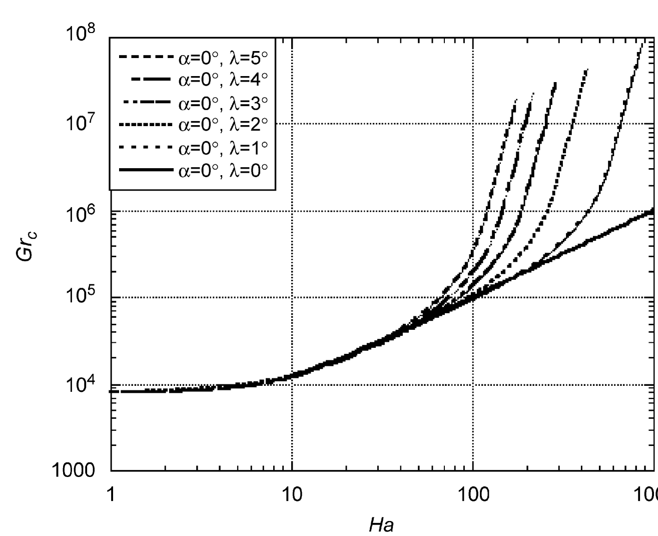

Fig. 2. Variation of Gr_c as a function of Ha for $Pr = 0.001$ and different values of λ.

values of Gr_c change significantly even when the deviation is as small as $\lambda = 1°$. In fact, when Ha becomes large ($Ha > 40$), the values of Gr_c deviate from the asymptotic behaviour, $Gr_c \sim Ha$, obtained for $\lambda = 0°$ and this occurs all the more early as the value of λ is large. Indeed, the vertical component induced by the weak deviation of the magnetic field direction is responsible for the two-dimensional modes stability improvement. As an example, taking $Ha = 150$, the threshold value which is $Gr_c = 152\,390$ for $\lambda = 0°$, becomes almost 40 times more, $Gr_c = 5\,995\,000$ for $\lambda = 5°$. For $Ha = 170$, the difference between the value of the critical Grashof number for $\lambda = 0°$ and $5°$ becomes more significant, with a ratio $Gr_c(\lambda = 5°)/Gr_c(\lambda = 0°) \sim 100$. However, these two-dimensional instability modes which were effective without limit in terms of Ha values for the longitudinal magnetic field disappear beyond a limit value of Ha when λ is non-zero. This limit value Ha_ℓ is 860 for $\lambda = 1°$, 429 for $\lambda = 2°$, 287 for $\lambda = 3°$, 215 for $\lambda = 4°$ and 173 for $\lambda = 5°$. From these results, a criterion for the existence of these two-dimensional modes under a longitudinal magnetic field with a weak polar deviation λ can easily be deduced as $Ha \sin \lambda < 15$. This result is consistent with those of Kaddeche et al. [12] who showed that under the action of a perfectly vertical magnetic field, the two-dimensional instabilities only exist for $Ha < 15$.

In Fig. 3 are displayed the curves of the wave number h_c as a function of Ha, for various values of λ. From the figure, we can note that for $Ha > 20$, with increasing Ha the wave number decreases following a nearly asymptotic behaviour, $h_c \sim Ha^{-1}$. However, such an asymptotic behaviour which means that the cell size becomes larger as Ha is increased, stops beyond a certain value of Ha and then the curves undergo a new behaviour. This particular value of Ha beyond which h_c starts to increase, appears to be a decreasing function of λ. Such a feature is due to the fact that when increasing the value of Ha, the vertical component of the magnetic field (perturbation component) progressively controls the marginal cell length.

3.1.2. *Weak deviation with respect to a vertical magnetic field*

In Fig. 4, the variation of the critical Grashof number as a function of α for $Ha = 15$ and three values of β (0°, 2.5° and 5°) is plotted. The figure shows clearly that Gr_c decreases when the magnetic field deviates from the perfectly vertical direction ($\beta = 0°$). The curves are symmetric with respect to $\alpha = 180°$ where Gr_c reaches its maximum value. From the results obtained for $0° \leqslant \alpha \leqslant 360°$, it is possible to show that the relative difference in terms of critical Grashof number with respect to the case $\beta = 0°$, defined as $\delta Gr_c = (Gr_c(\alpha = 0°, \beta = 0°) - Gr_c(\alpha, \beta))/Gr_c(\alpha = 0°, \beta = 0°)$, reaches its maximum value for $\alpha = 0°$ and its minimum value for $\alpha = 180°$. For example, the maximum value is $\delta Gr_{c,\max} = 4.115\%$ for $Ha = 15$ and $\beta = 5°$. In Fig. 5, where is depicted the wave number $K_c = \sqrt{h_c^2 + k_c^2}$ as a function of α, one can notice that the length of the marginal cells ($\ell_c = 2\pi/K_c$) becomes slightly narrower compared to the case $\beta = 0°$, with a maximum relative variation of the order of 0.086% reached for $\beta = 5°$ and $Ha = 15$ when $\alpha = 90°$ and $\alpha = 270°$. Furthermore, except for the cases $\alpha = 0°$, 180° and 360°, k_c (the wave number along $\overrightarrow{e_y}$) becomes different from zero indicating that the wave front becomes slightly tilted with respect to the streamwise direction. This behaviour can be attributed to the horizontal component of the magnetic field which generally tends to align the marginal cell axis with its own direction as discussed by Kaddeche et al. [19]. Let us mention that such an inclination with respect to the streamwise direction remains weak and corresponds to a maximum angle of 0.2012° reached for $Ha = 15$ and $\beta = 5°$ at $\alpha = 90°$ and 270°. Nevertheless, this modification of the

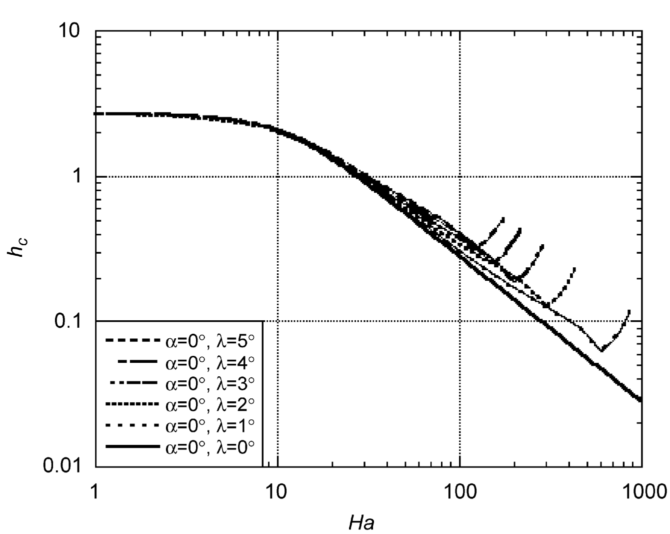

Fig. 3. Variation of h_c versus Ha for $Pr = 0.001$ and different values of λ.

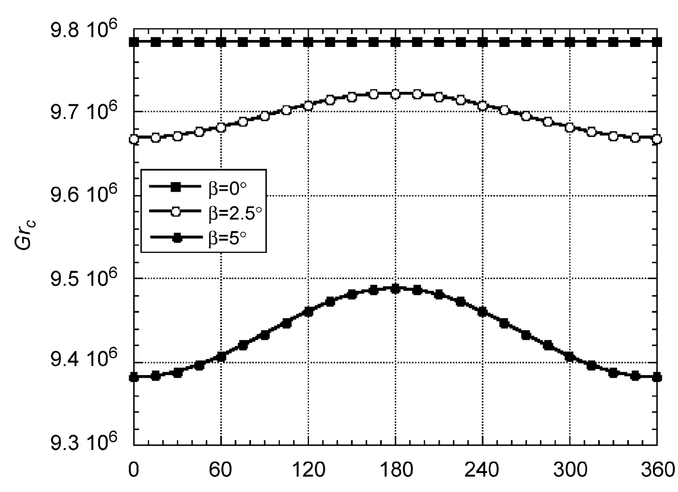

Fig. 4. Variation of Gr_c versus α for $Ha = 15$, $Pr = 0.001$ and different values of β.

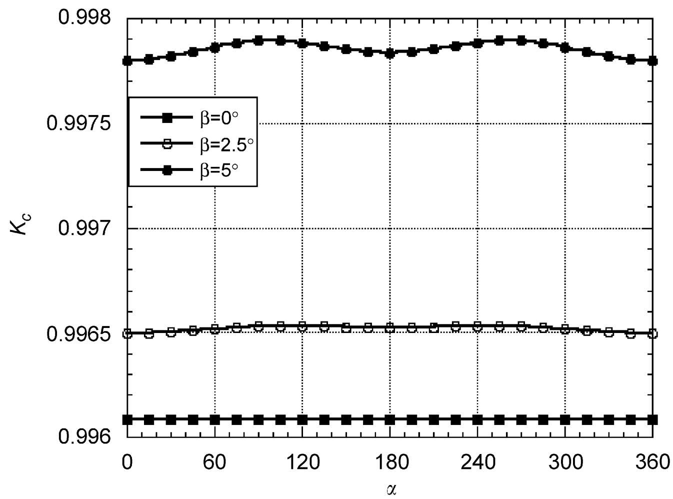

Fig. 5. Variation of K_c versus α for $Ha = 15$, $Pr = 0.001$ and different values of β.

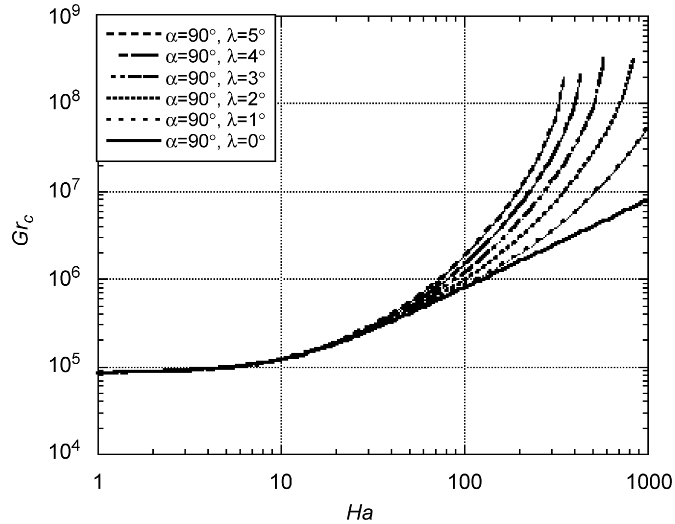

Fig. 6. Variation of Gr_c versus Ha for $Pr = 0.02$ and different values of λ.

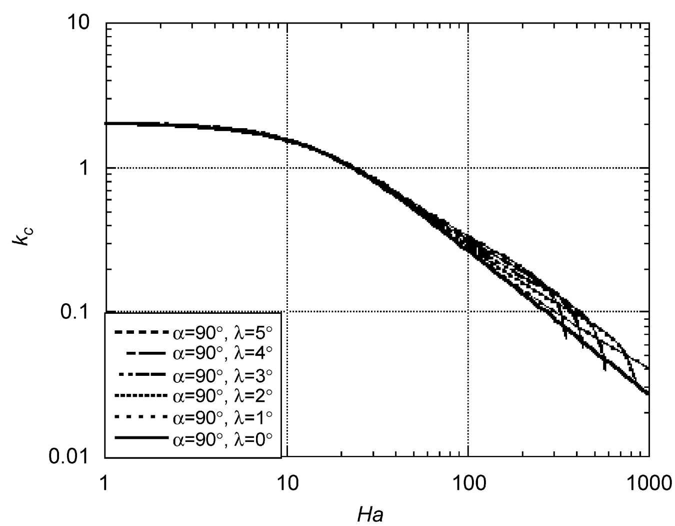

Fig. 7. Variation of k_c versus Ha for $Pr = 0.02$ and different values of λ.

mode which becomes slightly oblique, preserves its stationary character.

3.2. Effect of a weak polar deviation on the longitudinal instabilities

3.2.1. Weak deviation with respect to a transverse magnetic field

In a similar way, we consider the effect of a weak polar deviation of the applied transverse magnetic field ($\alpha = 90°, \beta = 90°$) on the stabilization of the three-dimensional longitudinal modes. As in the previous case, we consider that the deviation λ is limited to 5°. The considered value of the Prandtl number for this part is $Pr = 0.02$. In Fig. 6, the curves giving the variation of Gr_c as a function of Ha for various values of λ are plotted. In this figure, one can remark that for $\lambda = 0°$ and for $Ha > 20$, the critical Grashof number increases linearly with Ha. Moreover, the critical Grashof number increases with λ. For instance, at $Ha = 200$, the critical Grashof number increases with λ as $Gr_c \sim \exp(\frac{2}{5}\lambda)$. Furthermore, when λ increases, the evolution of the curves deviates more and more from the linear behaviour obtained at $\lambda = 0°$. We have to point out that, in this case also, the instabilities disappear beyond a limit value of Ha for λ non-zero. This limit value Ha_l is 1833 for $\lambda = 1°$, 917 for $\lambda = 2°$, 611 for $\lambda = 3°$, 459 for $\lambda = 4°$ and 367 for $\lambda = 5°$. From these results, a criterion for the existence of the three-dimensional modes under a transverse magnetic field with a weak polar deviation λ can easily be deduced as $Ha \sin \lambda < 32$. This result is still consistent with those of Kaddeche et al. [12] who showed that under the action of a perfectly vertical magnetic field, the three-dimensional instabilities exist only for $Ha < 32$. From Fig. 7, where is plotted the wave number k_c as a function of Ha for various values of λ, we note that k_c decreases with increasing Ha. For $Ha > 20$, the wave number undergoes an asymptotic behaviour, $k_c \sim Ha^{-1}$. Such an asymptotic behaviour which means that the cell size becomes larger as Ha is increased, stops beyond a certain value of Ha and then the curves undergo a new behaviour. This particular value of Ha beyond which k_c starts to decrease sharply, is a decreasing function of λ. This sharp decrease is connected to the vertical perturbation of the magnetic field, as such a decrease is found in the case of a perfectly vertical magnetic field in connection with the disappearance of the instability. From Fig. 8, one can notice that slight deviations with respect to the transverse magnetic field, affect significantly the values of the critical frequencies f_c, especially for $Ha > 30$. As it can be observed, for all the considered deviation values ($\lambda \leqslant 5°$) the frequency varies slightly in the range $Ha < 30$ and starts to grow quickly for larger values of Ha. Note that in this range of Ha values ($Ha > 30$), the frequency is constant at $\lambda = 0°$. Moreover, the sharp increase of the frequency occurs more and more early when the value of λ increases. As an example, for $Ha = 300$, f_c rises from the value 65.8

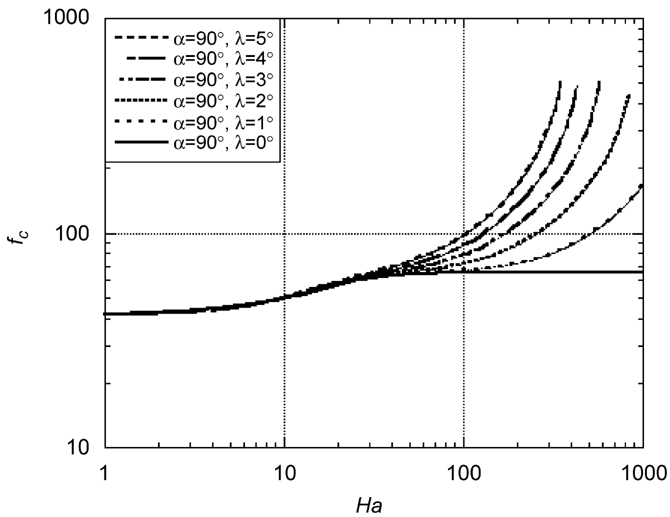

Fig. 8. Variation of f_c versus Ha for $Pr = 0.02$ and different values of λ.

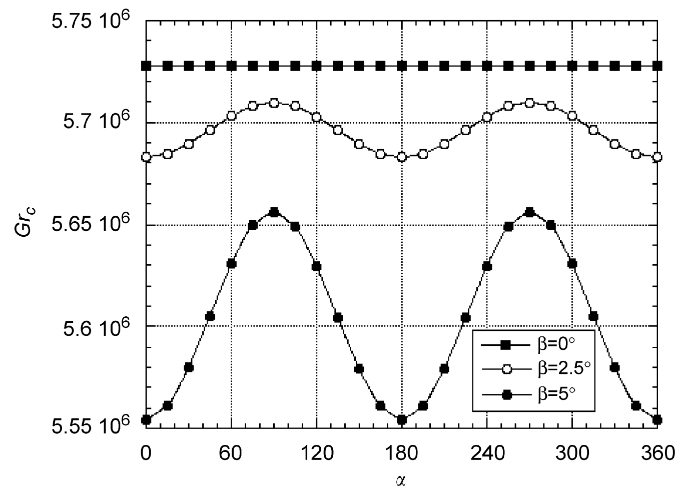

Fig. 9. Variation of Gr_c versus α for $Ha = 32$, $Pr = 0.02$ and different values of β.

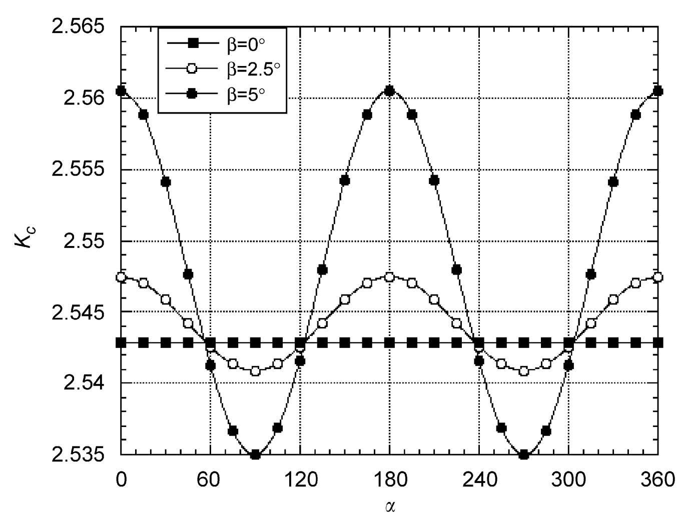

Fig. 10. Evolution of K_c versus α for $Ha = 32$, $Pr = 0.02$ and different values of β.

for $\lambda = 0°$ to the value 316.1 for $\lambda = 5°$. According to Refs. [10,12], one knows that the vertical magnetic field is responsible for the increase of the frequencies. More precisely, the frequencies which remain moderate for a perfectly transverse magnetic field ($40 < f_c < 70$) can reach values up to 400 for a perfectly vertical magnetic field and $Ha \sim 32$. This explains why the vertical perturbation of the magnetic field is responsible for this sharp increase of the critical frequencies when $Ha \sin\lambda \sim 32$. If this increase of the frequencies is sufficiently significant, it should have a positive impact on the grown crystal. Otherwise, if the frequency remains low or moderate, the crystalline end-product could be affected by undesirable striations.

3.2.2. Weak deviation with respect to a vertical magnetic field

In Fig. 9, we present the curves of Gr_c as a function of α for $\beta = 0°$, 2.5°, 5° and $Ha = 32$. From the figure, it is clear that the values of Gr_c decrease when β increases. Moreover, when β increases, the evolution of Gr_c as a function of α becomes increasingly important. For $\beta = 5°$, the maximum value of δGr_c is reached for $\alpha = 0°$, 180° and 360°. This maximum value $\delta Gr_{c,max}$ is equal to 3.028%. In Fig. 10, the curves of the wave number K_c as a function of α for $Ha = 32$ are displayed. One can notice that the wave length of the marginal cells varies significantly with α. Indeed, depending on the value of α, the size of the marginal cells can be larger or smaller than that corresponding to the case of the perfectly vertical magnetic field, whereas it remains narrower for the case of transverse instabilities. Nevertheless, the relative difference affecting the marginal cell length does not exceed 0.6912%. Furthermore, when α is not a multiple of 180°, h_c (the wave number along $\overrightarrow{e_x}$) becomes different from zero indicating that the wave front becomes slightly inclined with regard to the spanwise direction. As emphasized previously, the origin of this behaviour is attributed to the horizontal component of the magnetic field. Nevertheless, for $Ha = 32$, the maximum deviation of the wave front with respect to the spanwise direction remains weak and corresponds to a maximum angle of 0.4386° reached for $\alpha \sim 105°$ and $\beta = 5°$. In Fig. 11, the variation of the critical frequency as a function of α for $\beta = 0°$, 2.5°, 5° and $Ha = 32$ is illustrated. The largest value of the frequency corresponds to 426.8 for $\beta = 0°$ and the differences with this case for all the non-zero β values, are quite moderate and do not exceed 2.053%, value obtained for $\alpha = 0°$ and 180°. Note that in this case, symmetries with respect to both $\alpha = 180°$ and 90° are observed for all the critical characteristics Gr_c, K_c and f_c.

4. Concluding remarks

By means of a linear stability analysis, the effects of a weak polar deviation of a longitudinal, transverse or vertical magnetic field on both two-dimensional stationary

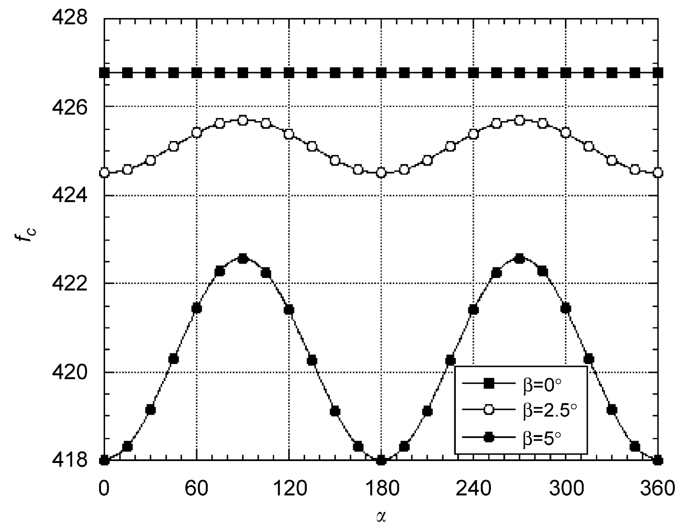

Fig. 11. Evolution of f_c versus α for $Ha = 32$, $Pr = 0.02$ and different values of β.

and three-dimensional oscillatory instabilities which develop in a differentially heated liquid layer bounded by horizontal rigid walls, were investigated. From the obtained results, we can state that some differences exist between the behaviour of the two types of instability under a weak polar deviation of the magnetic field. For the same deviation values, the three-dimensional instabilities are less stabilized than the two-dimensional ones. Such a result can be connected to the fact that the most efficient component of the applied magnetic field is the vertical one and it better stabilizes the two-dimensional transverse instabilities than the three-dimensional longitudinal modes. Besides, even a weak polar deviation value, $\lambda \leqslant 5^\circ$, produces remarkable changes in the stabilization of both two-dimensional stationary modes when a longitudinal magnetic field is considered and three-dimensional oscillatory modes when a transverse magnetic field is considered. In fact, the critical Grashof number Gr_c and the wave numbers h_c and k_c do not follow any asymptotic tendency for large values of Ha, contrarily to the case without magnetic field deviation. In addition, the curves giving the variation of the critical Grashof number deviate more and more from the asymptotic behaviour found for $\lambda = 0^\circ$ since Gr_c grows more rapidly with Ha as the value of λ increases. Such a behaviour is found to be more marked for the two-dimensional stationary modes than for the three-dimensional oscillatory modes. The ranges of Hartmann number where both instabilities exist are $Ha \sin \lambda < 15$ for the two-dimensional modes and $Ha \sin \lambda < 32$ for the three-dimensional modes. Moreover, for three-dimensional oscillatory modes, the polar deviation of the magnetic field enhances the growth of the critical frequency curves which start to sharply increase when Ha exceeds some particular value, found to be a function of the deviation λ. This result may have consequences on the homogeneity of the grown crystal.

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